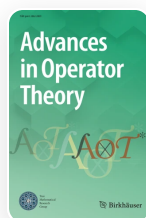


Asymptotics of the eigenvalues of seven-diagonal Toeplitz matrices of a special form

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Abstract

This work is devoted to the construction of a uniform asymptotics in the dimension of the matrix n tending to infinity of all eigenvalues in the case of a seven-diagonal Toeplitz matrix with a symbol having a zero of the sixth order, while the cases of symbols with zeros of the second and fourth orders were considered earlier. On the other hand, the results obtained refine the results of the classical work of Parter and Widom on the asymptotics of the extreme eigenvalues. We also note that the obtained formulas showed high computational efficiency both in sense of accuracy (already for relatively small values of n) and in sense of speed.



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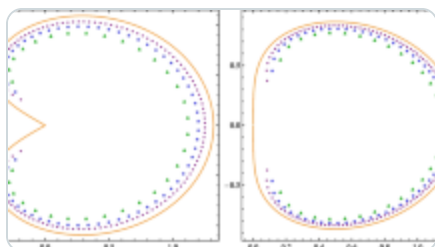
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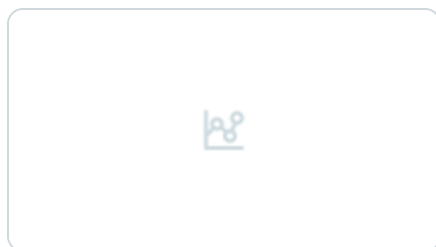
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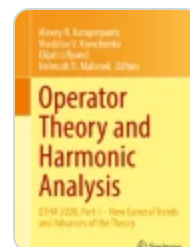
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Additional information

Communicated by Estelle Basor.

Dedicated to Ilya Spitkovsky on his occasion of his 70th birthday.

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